MODELLING DIFFUSION OF INNOVATIONS WITH HOMOGENEOUS AND HETEROGENEOUS POPULATIONS

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ABSTRACT

Diffusion of innovation can be defined as a system of adoption by which the innovation of a new product or service is spread among the members or agents in a social system. Diffusion models help to forecast the demand and work as a decision aid in making pre-launch, launch and help post-launch strategic choices. This complex system can be approached, in aggregate form, as a function of the number of adopting agents or the number of adoptions, as a function of time and controlling covariates or as a function of the time and spatial aspects simultaneously in the related structure. The process can also be described with simultaneous and dynamic models depending upon whether innovations are independent, complementary or substitutes. The present study offers a framework for systematising the diffusion models in the perspective of existing homogeneity or heterogeneity among the agents. Different models are compared, new models are proposed and their advantages and disadvantages discussed with reference to the Algerian natural gas production time series. Some guidelines for further research extensions are also suggested.

1. INTRODUCTION

In recent decades, the pattern of diffusion of innovations for products or services has become an interesting matter of study for social scientists, mathematicians, marketing experts, statisticians, engineers and biologists. Researchers are drawn to the topic not only to examine trends and underlying factors in the diffusion process but also for forecasting purposes. In this context, researchers try to understand the behaviour of the existing individuals (agents) in society and their attitude towards newly introduced goods or services and explain them with special mathematical models. This attitude can be termed “adoption” the marketing language that divides the existing non-homogeneous agents into several mutually exclusive groups. Bass (1969) considers two sub-populations of adopters, innovators and imitators, and develops an aggregate model for the adoption assuming homogeneity in the sub-population units. This paradigm of innovation diffusion modelling proceeds with further diversification of understanding the process also followed by a number of studies and reviews (see Mahajan et al. (1990), Bass (2004), Meade and Islam (2006)). The important thing here is to model the social contagion and adoption of goods/services in a social system that is characterised by specific regime changes that cumulatively follow a sigmoid shape. This system of adoption also depends
on the structure of the social system, on its internal rules or external influences (policies, marketing strategies etc.) that may be considered to vary with respect to time. Rogers and Shoemaker (1971) define the diffusion of innovation as the process by which innovation spreads among the members of a social system. The innovation itself, adopters of the innovation, innovation channels, time and space, change agents and the social system dynamics seem to be associated with this process. Various modelling approaches were followed by the researchers to obtain time patterns of the diffusion process. The fundamental approach is to consider the diffusion process as a direct function of time. The alternative approach is to consider the process as a function of the number of previous adopters over time through special differential equations which may include theoretical assumptions and therefore, easy-to-interpret parameters. Others extended the fundamental diffusion model to study the time and spatial aspects of the diffusion process simultaneously. Some researchers also attempted to model a simultaneous and dynamic diffusion model, depending upon whether innovations are independent, complementary or substitutes.

The objective of the present study is to conduct an in-depth study of the existing diffusion of innovation models with both homogeneity and heterogeneity assumptions in the population and make a valid comparison of their parameter estimates. In Section 2, a short discussion on the standard Bass model and its extensions is presented. Section 3 contains the discussion of diffusion of innovation models appropriate for a heterogeneous population with special emphasis on Gamma-shifted Gompertz models and their extensions. Model parameter estimates and their comparisons are discussed in Section 4. Section 5 is devoted to the analysis of the Algerian natural gas production within the logic of a diffusion of innovation process. Some interesting results are provided with reference to the evolving dynamics, peak time and reserves estimation. Finally, an overall discussion and further extension guidelines are presented in Section 6.

2. THE BASS MODEL: A PARADIGM OF A HOMOGENEOUS DIFFUSION MODEL

The fundamental diffusion model by Bass (1969) is based on the assumptions that the probability of adoption of a new product or innovation at time $t$ given that it has not yet been adopted would depend on a convex combination of two factors: the number of independent initial adopters and the number of existing adopters (i.e., imitators). The innovation coefficient measures the propensity of potential adopters to become adopters, and the imitation coefficient measures the propensity of potential adopters to imitate previous adopters. The Bass model was built on Roger’s conceptual framework by developing a mathematical model that captures the non-linear structure of the S-shaped curve (Robertson et al. (2007)). Introducing the coefficient of innovation $p$ ($p>0$) and the coefficient of imitation $q$ ($q>0$), the Bass model can be described by the following equation:

$$f(t) = [p + qF(t)][1 - F(t)]$$  \hspace{1cm} (1)

where $F(t)$ depicts the distribution over time of adoptions and $f(t)=F(t)$ denotes the corresponding density of the adoption process over time. Under the initial condition $F(0)=0$, its solution (Bass (1969)) defines the following distribution function:
Let $m$ be the number of potential adopters (or adoptions) in the market. Then the total number of adoptions until time $t$ is therefore obtained as follows:

$$Y(t) = mF(t) = m\left(1 - \frac{q}{p} e^{-(p+q)t}\right)$$

Considering the time unit as unity (year, quarter, month, week, days etc.), the rate of diffusion, in other words, let’s say, sales $S(t)$ in the time interval $(t-1,t)$, is given by the following:

$$S(t) = mf(t) + \varepsilon(t)$$

$$\approx m[F(t) - F(t-1)] + \varepsilon(t)$$

$$= m\left[1 - e^{-(p+q)t} - 1 - e^{-(p+q)(t-1)}\right] + \varepsilon(t)$$

Considering $\varepsilon(t)$, the error term in the equation as distributed with variance $\sigma^2$, the parameters $p$, $q$ and $m$ can be estimated by the non-linear least squares (NLS) procedure (Srinivasan and Mason (1986)).

A better approximation for $f(t) = F'(t)$ can also be obtained through the following:

$$f(t) \approx F(t + 0.5) - F(t - 0.5)$$

The Bass model is the first and foremost formal way to separate the innovators (leaders) and imitators (followers) in an innovation process that better explains Roger’s perspective based on a normal distribution assumption. Innovators and imitators characterise a latent distinction, since the observed data just report on the adoption of a susceptible agent without any other specification.

A very important extension of the Bass model developed by Bass et al. (1994) is based on the Generalised Bass Model (GBM), which introduces a general time dependent intervention function $x(t)$, able to take into account the possible effect of the exogenous variables on the diffusion process. Thus, an extension of Equation (1), under the initial condition $F(0)=0$, is given by the following:

$$f(t) = [1 - F(t)](p + qF(t))x(t)$$

and its solution gives an expression for the total number of adopters until time $t$ as follows:

$$Y(t) = m\left\{1 - e^{-(p+q)\int_0^t x(\tau)d\tau}\right\} = mF(t), \quad 0 \leq t \leq +\infty$$

$$\left\{\begin{array}{l}
1 - e^{-\int_0^t x(\tau)d\tau} \\
1 + \frac{q}{p} e^{-\int_0^t x(\tau)d\tau}
\end{array}\right\}$$
Bass et al. (1994) called this function $x(t)$ the “current marketing effort” that reflects the current effect of dynamic marketing variables on the conditional probability of adoption at time $t$. Notice that the closed-form solution (4) is extremely general, because the control function $x(t)$ may assume, under local integrability, any shape without special limitations. For $x(t)=1$, the model reduces to the standard Bass model, and for $x(t)>1$, the adoption process is accelerated over time; otherwise, it is delayed (Guseo et al. (2007) and Dalla Valle & Furlan (2011)). Therefore, this intervention function may modify the time elapsing between adoption events within a general closed-form solution very powerfully in applied contexts.

The basic Bass model fits very well to real data, and many other versions of the model appeared later to explain different aspects of diffusion. A special application of the GBM has been made in the energy sector, crude oil in particular (Guseo & Dalla Valle (2005) and Guseo et al. (2007), Guseo (2011)) where the rationale for these applications is grounded on the related diffusion technologies that are directly or indirectly energy consuming.

Guseo et al. (2007) model the intervention function $x(t)$ through some exponential shocks, under the assumption that the memory effect has a non-uniform distribution over time. Thus, the function $x(t)$ can be defined by the following:

$$x(t) = 1 + c_1 e^{a_1 (t-a_1)} I_{t \geq a_1} + c_2 e^{b_1 (t-a_2)} I_{t \geq a_2}$$

where $a_i (i=1,2)$ denotes the starting times of exponential shocks, $b_i (i=1,2)$ describes the effect’s persistence and $c_i (i=1,2)$ controls the intensity of perturbations. For the values of parameter $b_i<0$, the process is mean reverting (i.e., the memory is decaying to the stationary position; in other words, $x(t)=1$). If $b_i>0$, the process introduces a permanent acceleration in the saturation of a life cycle.

Despite recent developments, the Bass model still suffers conceptual limitations in applications and forecasting. It assumes that the internal influence (word-of-mouth effect) remains uniform over time frame over the diffusion process period. Conversely, in practice, the later adopters are not as likely to discuss the product with non-adopters as are the early adopters, and they are also less likely to exhibit the same enthusiasm in discussing the new product. In other situations, the internal influence becomes increased due to the influence of the reluctance of later adopters to the word-of-mouth effects. In many occurrences, the late adopters have different characters than the early adopters and would respond differently (Rogers (2003)) and the diffusion model should allow for this. Therefore, researchers have suggested a number of alternative structures to the intervention function $x(t)$ to comply with the existing population structures modelled with the shocks as exponential or rectangular or both types in the observed dataset.

Bass models, BM and GBM, have a fixed market potential over the assumed life cycle. An important extension, the dynamic market potential, $m(t)$, is introduced in Guseo (2004) and Guseo and Guidolin (2009, 2010, 2011). In particular, Guseo and Guidolin (2009) obtain a Riccati closed-form solution for general $m(t)$ and $x(t)$ functions that emphasizes the different role of policies over time $x(t)$ and over scale $m(t)$ in order to describe the time modulation of a non-constant carrying capacity (market potential).
3. A NON-HOMOGENOUS DIFFUSION MODEL: THE GAMMA SHIFTED GOMPERTZ MODEL

In recent days, considering the heterogeneity of the population, a limited number of diffusion models have been introduced that incorporate individual-level heterogeneity and/or heterogeneity in the diffusion penetration rate. Researchers try to develop a segmental diffusion model (Robertson et al. 2007), or models considering several distributional assumptions of market penetration rate and adoption at the individual level (Bemmaor (1994); Van den Bulte and Lilien (1997); Gutierrez-jaimez et al. (2007)). The principal matter of interest in this case is to obtain a parsimonious and flexible closed-form diffusion model that can accommodate both symmetric and non-symmetric diffusion patterns with a point of inflection that can occur at any stage of the diffusion process (Mahajan and Wind (1986)).

The research paradigm on diffusion of innovation in a social system by Bass (1969) and Mansfield (1961) and their generalisations addressed the market as an aggregate structure, with little attention to micro-level processes that characterise adoption decisions (Chatterjee and Eliashberg (1990); Mahajan et al. (1990)). The main issue in this line is to understand and explain the diffusion process across a population of adopting units. The existence of a heterogeneous population of adopters has been largely ignored in this perspective. In the individual-level perspective, the diffusion of innovation can either be modelled as individual adoption probability with the timing of adoption or derivation of adoption behaviour at the individual level in a decision-theoretic framework (Chatterjee and Eliashberg (1990); Rose and Joskow (1990); Sinha and Chandrashekaran (1992)).

The model by Chatterjee and Eliashberg (1990) considers the heterogeneity in initial perceptions of the innovation’s performance, consumers’ preference structure and the perceived reliability of information on which updating takes place. Their approach is an important step in modelling diffusion at the micro level, but its application is limited by its dependence on extensive perceptual data about adoption. Sinha and Chandrashekaran (1992) use a hazard model approach that explicitly incorporates covariates in the adoption time specification so that population is heterogeneous in timing of adoption. The considered split hazard model framework allows for modelling the adoption decision at the individual level as well as describing and forecasting new product acceptance at the aggregate market level.

Bemmaor (1994) suggests an alternative approach to explain the changes in the parameter estimates of the Bass model considering the underlying heterogeneity of the population. He considers that diffusion can equivalently be explained by the variation of individual propensities to buy across consumers. Therefore, a shifted Gompertz density can explain the timing of first purchase, and the individual propensity across consumers follows a gamma distribution. The aggregate diffusion process results in a mixture of these two densities. Bemmaor and Lee (2002) briefly analysed the consequence misspecification in the Bass model mentioned by Van den Bulte and Lilien (1997) and found the supremacy of the Bemmaor (1994) model with strong forecasting capacity. Their results also suggested that the Gamma-shifted Gompertz model is a flexible model.
to analyse the systematic changes in parameter estimates when specification error and ill-conditioning occur.

The Gamma-shifted Gompertz model postulates that the ratio $q/p$ of Bass model parameters varies with scale parameter $\beta$ of the heterogeneity distribution of $\eta$ and $\eta$ has to be distributed according to a Gamma $(\alpha, \beta)$ law. Therefore, the individual level model (model of first adoption timing) can be identified with a shifted Gompertz density as follows:

$$ F(t | \eta, b) = (1 - e^{-bt}) e^{-\eta e^{-bt}}, \quad t > 0, \quad \eta, b > 0 $$

with the following density function:

$$ f(t | \eta, b) = be^{\eta e^{-bt} - bt - \eta} [1 + \eta(1 - e^{-bt})], \quad t > 0, \quad \eta, b > 0 $$

For a fixed value of $b$, the small values $\eta$ imply a low mean time of adoption (i.e., a strong individual propensity to buy).

If the heterogeneity parameter $\eta$ varies according to a Gamma distribution with shape parameter $\alpha$ and scale parameter $\beta$, the aggregate-level diffusion model can be described by the following cumulative distribution function:

$$ F(t) = (1 - e^{-bt}) / (1 + \beta e^{-bt})^\alpha $$

(5)

With the following density function:

$$ f(t) = be^{-bt} (1 + \beta e^{-bt})^{-(\alpha+1)} [1 + \alpha \beta + \beta e^{-bt} (1 - \alpha)] $$

If we re-parameterise Equation (3) with the equivalent Bass model coefficient by letting $b = p+q$ and $\beta = q/p$, we obtain the following aggregate-level diffusion model:

$$ F(t) = \left(1 - e^{-(p+q)t}\right) \left(1 + \frac{q}{p} e^{-(p+q)t}\right)^\alpha, \quad t \geq 0 $$

(6)

For the value of shape parameter $\alpha=1$, the model reduces to the standard Bass model. When $\alpha=0$, the model reduces to an exponential model, and when $\alpha=\infty$, it converges to the shifted Gompertz model (Mahajan and Peterson (1985)). Therefore, as $\alpha$ approaches to zero, the shape of the diffusion curve resembles an exponential diffusion curve, and for larger values of $\alpha$, it approaches a logistic curve. The role of $\alpha$ becomes the vital driver in the diffusion process that can be used to explain the impact of contagion or lack of contagion in the diffusion process. Thus, the Gamma-shifted Gompertz model comprehends several other models of diffusion, including the standard Bass model.

The Gamma-shifted Gompertz model by Bemmaor (1994) is an important contribution in diffusion modelling that provides grounds to investigate jointly “the speed takeoff” and “the diffusion speed after takeoff” observed in the process. As all other diffusion patterns are nested within the Bemmaor modelling approach, our idea is to extend it and make this model able to incorporate the related exogenous variables in the diffusion process considering the modifications with the GBM approach, incorporating the intervention function $x(t)$.

Considering an intervention to the Bemmaor model with one exponential shock that starts at time $a$, with intensity $c$ and persistent effect $b$, a GBM Bemmaor model can be given by the following equation:
\[ F(t) = \begin{cases} 
\left\{1 - e^{-(p+q)t}\right\}^{\gamma} / \left\{1 + \frac{q}{p} e^{-(p+q)t}\right\}^\alpha & ; \ t < a \\
\left\{1 - e^{-(p+q)(t-a)}\right\}^{\gamma} / \left\{1 + \frac{q}{p} e^{-(p+q)(t-a)}\right\}^\alpha & ; \ t \geq a 
\end{cases} \]

(7)

It should be noted that the Bemmaor model has two main portions: the numerator that explains the influence of innovators and the denominator that explains the effect of imitators on the ultimate penetration. Therefore, it can be applied to model (6) with a simple modification after introducing a non-negative exponent to the innovators’ influence that gives the expression for the following model:

\[ F(t) = (1 - e^{-(p+q)t})^{\gamma} / \left(1 + \frac{q}{p} e^{-(p+q)t}\right)^\alpha. \quad t \geq 0 \]

(8)

The new parameter \( \delta \) will speed up/suppress the initial start to the curve and modify the curve peakedness. This intervention of the modified model is important to describe the quick/delayed entrance of the innovators, which could be mixed up with other contagions processes. With a fixed \( \alpha \), for \( \delta=1 \), the modified model becomes the standard Bemmaor model, and \( \delta<1 \) will delay the innovators’ contagion process, whereas \( \delta>1 \) will speed up the diffusion at the very beginning. In other words, the parameter \( \delta \) can be considered a measure of the propensity of the innovators to participate in the adoption process. For an ideal situation, \( \delta \) should be greater than \( \alpha \), and for \( \delta=\alpha=1 \), the proposed modified model equals the standard Bass model.

The modified Bemmaor model can also be used with further modifications by adding some exponential or rectangular shocks. An equivalent expression as the model in Equation (7), the modified Bemmaor model with intervention function can be given by the following cumulative distribution function:

\[ F(t) = \begin{cases} 
\left\{1 - e^{-(p+q)t}\right\}^{\gamma} / \left\{1 + \frac{q}{p} e^{-(p+q)t}\right\}^\alpha & ; \ t < a \\
\left\{1 - e^{-(p+q)(t-a)}\right\}^{\gamma} / \left\{1 + \frac{q}{p} e^{-(p+q)(t-a)}\right\}^\alpha & ; \ t \geq a 
\end{cases} \]

(9)

These modified models in Equations (7-9) appearing for the first time, are very simple but important in terms of explaining the contagion effect with specific parameterisation of innovators’ and imitators’ penetration in the diffusion process. The analytical validation of the above postulates will be discussed in Section 5 with a real dataset.

4. MODEL PARAMETER ESTIMATES AND INFERENCE

The GBM in Equation (4) and the heterogeneity models in Equations (6-9) can be specified in a non-linear regressive framework as follows:

\[ z(t) = f(\beta, t) + \epsilon(t) \]

(10)

where \( z(t) \) represents the cumulative observed data, \( f(\beta, t) \) is the deterministic component of the model specified through the cumulative mean function of \( f(t) \) of adoption over time, \( \beta \) is the vector of parameters and \( \epsilon(t) \) is a white noise process. The model parameters can be estimated using the NLS method following the Levenberg-Marquardt algorithm (Seber and Wild (1989)). At the second step, the estimated function \( f(\hat{\beta}, t) \) can be used in an ARMAX model in order to obtain a convenient expression of
the residual structure in $\varepsilon(t)$ that may be characterised by auto dependence effects very far from a standard white noise.

Following Guseo et al. (2007), the significance of the gain from the simpler model to the more complex model can be evaluated in two steps. As a first step, the squared multiple partial correlation coefficient is computed by the following:

$$\tilde{R}^2 = \left( R^2_{M_1} - R^2_{M_2} \right) / \left(1 - R^2_{M_2} \right)$$  \hspace{1cm} (11)

where $R^2_{M_2}$ denotes the determination index of the reduced model that has to be compared to model $M_1$. If $N$ denotes the total number of observations used to fit the models, and $\lambda$ is the number of parameters included in model $M_1$, the significance for the number of $\kappa$ parameters of the model $M_1$ that are not included in model $M_2$ can be evaluated by a special form of F-statistics defined as follows:

$$F = \left[ \tilde{R}^2 (N - \lambda) \right] / \left[ (1 - \tilde{R}^2) \kappa \right]$$  \hspace{1cm} (12)

which is distributed as a Snedecor’s-$F$ with $\{ \kappa, (N - \lambda) \}$ degrees of freedom under the assumption of equivalence of models $M_1$ and $M_2$ with $\varepsilon(t)$ is normal i.i.d. Considering the common threshold 4 for the F-ratio in (12) as an approximate robust criterion to compare model $M_2$ nested in model $M_1$, the comparative performance can be evaluated (Guseo et al. (2007)).

5. AN APPLICATION TO THE ALGERIAN NATURAL GAS PRODUCTION DATA

Algeria has one of the biggest natural gas reserves in the world. Algeria is the owner of the eighth-largest natural gas reserve, having 159 trillion cubic feet (TCF) of proven natural gas, according to Oil and Gas journal. Results from the BP Statistical Review of World Energy 2010 indicate Algeria as the holder of 2.4% of the total world gas reserves. The reserve-to-production ratio is 55.3 years, but this type of index is often questioned, because it does not take into account the nonlinear extraction dynamics. The country is the third-largest exporter of natural gas to Europe. Algeria’s natural gas sector has witnessed rapid expansion on the heels of increased production. Recent successes are aided by the international partnerships and technological advances, and the country is, at the same time, looking forward to solidifying its standing as a regional transit hub for natural gas, Global Arab Network reports according to Oxford Business Group (OBG). ‘Sonatrach’ dominates the country’s natural gas production and wholesale distribution; however, foreign investments in the sector are continuously increasing. Foreign producers such as PCI, BP, Statoil, Total, BHP-Billiton, Eni and Repsol have entered into partnership agreements with ‘Sonatrach’ from the early 1970s.

The present study uses the Algerian natural gas production (in billion cubic meters, BCM) data obtained from British Petroleum (2011) for the period from 1970 to 2010. As seen in Figure-1, starting from the 1970’s, the scenario of Algerian gas production follows an increasing trend until 2005 with some ups and downs at different sections but a slow decreasing trend afterwards. The increment trend is due to an increasing demand from the three top consumers (i.e. Italy, Spain and France), but the unexpected slow decrement trend after 2005 is a matter of discussion.
Figure-1: Natural Gas Production in Algeria (Instantaneous Data)
[Source: British Petroleum (2011)]

To describe the cumulative annual Algerian natural gas production, the study considers the model described in the previous section in Equations (2), (4), (6) and (7) with different specifications. Starting with the standard Bass (B1) model, it considers a GBM with an exponential shock for the intervention function $x(t)$. Afterwards, considering the heterogeneity, the standard Bemmaor model (BM) and Bemmaor model with GBM (GBMBM) are considered. A further generalisation as in Equations (8-9), modified Bemmaor model (BMM) and modified Bemmaor model with intervention function (GBMBMM), is also taken into consideration to identify the best fitted model and validate parameters. Obtained results are given in Table 1.

Table 1: Parameter Estimates and Asymptotic Standard Errors (In Parentheses) for Different Models

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>B1</th>
<th>BM</th>
<th>BMM</th>
<th>GBM</th>
<th>GBMBM</th>
<th>GBMBMM</th>
</tr>
</thead>
<tbody>
<tr>
<td>General penetration</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parameters m</td>
<td>2687 (59.71)</td>
<td>4948 (437.989)</td>
<td>3029 (184.45)</td>
<td>2993 (64.129)</td>
<td>3057 (267.21)</td>
<td>2833 (189.288)</td>
</tr>
<tr>
<td>p</td>
<td>0.0018 (0.00004)</td>
<td>0.0379 (0.00128)</td>
<td>0.0013* (0.00066)</td>
<td>0.0012 (0.00005)</td>
<td>0.00092* (0.0018)</td>
<td>0.00068* (0.00081)</td>
</tr>
<tr>
<td>q</td>
<td>0.124 (0.002469)</td>
<td>0.0096* (0.00528)</td>
<td>0.1155 (0.0151)</td>
<td>0.1118 (0.0025)</td>
<td>0.1099 (0.0269)</td>
<td>0.1298 (0.0241)</td>
</tr>
<tr>
<td>Exponential</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>shock parameters a</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>11.42 (0.4132)</td>
<td>9.44 (0.6602)</td>
<td>12.75 (0.5334)</td>
</tr>
<tr>
<td>b</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>-0.264 (0.0483)</td>
<td>-0.178 (0.8083)</td>
<td>-0.255* (0.1472)</td>
</tr>
<tr>
<td>c</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>1.158 (0.1485)</td>
<td>1.202 (0.4949)</td>
<td>0.6384 (0.1535)</td>
</tr>
<tr>
<td>Propensity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>parameters α</td>
<td>22.17 (9.7556)</td>
<td>0.763 (0.1103)</td>
<td>---</td>
<td>0.971 (0.4865)</td>
<td>0.756 (0.2158)</td>
<td>---</td>
</tr>
<tr>
<td>δ</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>3.233 (0.4525)</td>
<td>---</td>
<td>2.218 (0.4614)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.999407</td>
<td>0.999877</td>
<td>0.999907</td>
<td>0.999924</td>
<td>0.999923</td>
<td>0.999946</td>
</tr>
<tr>
<td>Model SE</td>
<td>14.9322</td>
<td>6.81086</td>
<td>5.9028</td>
<td>5.33395</td>
<td>5.38814</td>
<td>4.50681</td>
</tr>
</tbody>
</table>

B1: Standard Bass model.
BM: Bemmaor model.
BMM: Modified Bemmaor model.
GBM: Generalised Bass model with one exponential shock.
GBMBM: Bemmaor model with GBM.
GBMBMM: Modified Bemmaor model with intervention function.*Parameter not significant.

As shown in Table-1, the analysis results from the Algerian gas production dataset prove the efficacy of the newly introduced modification of the existing Bemmaor model with respect to the parameter estimates and respective fitness of the model. Compared to
the standard Bass model (B1) or Bemmaor model (BM), the modified Bemmaor models (BMM, GBMBM and GBMBMM) reach better $R^2$ values. Parameter estimates for $m$, the carrying capacity, associated with the limiting behaviour of the cumulative production process and represent a current estimate of the Ultimate Recoverable Resources (URR). All the above models except BM suggest that the natural gas production crossed the middle of the life cycle and the maximum production level was already reached.

The standard Bass model (B1) predicts a moderate net reserve for net natural gas reserve and shows a very slow contribution to the innovators and comparatively large decrements of imitators’ contribution to the process. The $R^2$ indicates the requirements for further modification of the fitted model. Since 2011, Algeria has produced 1921 BCM of natural gas, according to the Bass model, only 28% of the total reserve remains for the future. The GBM with one exponential shock shows a better fit to the data. It shows a mean-reverting positive shock around 1981/1982 when the Algerian state-dominated oil and gas company commissioned the Sonatrach Skikda LNG plant and refinery (GL-1K complex) and the government signed a 20-year agreement with France.

The Bemmaor model (BM) and the Bemmaor model with intervention function (GBMBBM) improve the model fitness in terms of $R^2$ and estimated standard error. A large value of the additional heterogeneity parameter in BM indicates the existing heterogeneity in the annual gas production and therefore describes the possibility for explaining the observed process in a shifted Gompertz setup. The BM model indicates that 61% of total natural gas is still unused. The GBMBBM, on the other hand, indicates the suitability of the usual GBM model with an estimate for the heterogeneity parameter of approximately 1 and an observed positive exponential shock in gas production around the 1980s that was absorbed in time.

The Modified Bemmaor model (BMM) and the modified Bemmaor model with intervention function (GBMBBMM) proposed in this study are completely new in the literature and include one additional parameter for the innovators’ heterogeneous propensity. Both models fit well with this additional parameter in terms of $R^2$ compared to all other considered models. The models have somewhat similar values for the imitators’ propensity level. The large innovators’ propensity coefficients for the BMM models indicate the existence of heterogeneity among initial productions, and a very accelerated trend with a late start is observed at the early stage of the diffusion process. The predicted reserve level is comparatively better for GBMBBMM, indicating that the maximum level of production was already reached and 67.8% of the Algerian natural gas URR had been extracted by 2011. The process also shows a positive mean-reverting exponential shock around 1983, when Algeria signed another gas export agreement with Italy and the first BTUs of gas were delivered through the Transmed pipeline.

Finally, based upon the comparative parameter estimates, model standard errors and $R^2$ values help to select the best fitted model among the postulated models. In all respects, the GBMBBMM model performs better for describing natural gas production. To obtain an improved short-term prediction for the regressive approach of the postulated models, an ARMAX model, based upon one regressor or more lagged regressors depending upon the predictive values of the first regressive step, was implemented. Obtained forecasts for the
different models are given in Figures (2-7). Results obtained for model estimation and forecast performance are described in Table 2.

Figures (2–4) show the graphs for the observed Algerian natural gas production and the forecast with the standard Bass model (B1), Bemmaor model (BM) and modified Bass model (BMM) respectively with a convenient ARMAX sharpening for a better short-term prediction. The results show that the Algerian natural gas production already crossed the maximum production level in the year 2000 by B1 and BM predictions, whereas the modified Bemmaor model (BMM) indicated that maximum production was reached in 2006. The BM model shows a recovery trend and forecast another peak production level between 2012 and 2016. Therefore, the predicted life cycle becomes a little longer. Other models do not support this prediction, and a decrement trend is observed after maximum production in 2000 with some stationarity in the process.
Model forecasts from the Algerian gas production data with the GBM with an intervention function of one exponential shock, described by GBM, GBMBM and GBMBMM are shown in Figures (5-7). Results show that the maximum production level had already been achieved in 2006, which is different from the year predicted by the standard Bass model (B1) or Bemmaor model (BM) due to the consideration of the existing exponential shock in the data and its consequence after the strategic and planning decisions taken by the respective authority. A rapidly decreasing production process was also predicted by GBMBMM followed by the respective prediction with GBM and GBMBM set up with ARMAX corrections for short-term prediction.
Results from Table 2 indicate that both the root-mean squared error (RMSE) and mean-absolute prediction error (MAPE) attain minimum values for the GBMBMM model after the forecast with ARIMA (2,1) set-up when regressed with the predicted estimates. When compared with the standard Bass model (B1), the significance for the inclusion of additional parameter/s passes the F-test for all other postulated models. Similar results are also found for the parameter/s when compared with the Bemmaor model (BM). For the GBMBMM model, the squared partial correlation co-efficient $\hat{R}^2 = 0.9089$ (F=65.85), $\hat{R}^2=0.5610$ (F=14.483) and $\hat{R}^2=0.2895$ (F=6.723) when compared with B1, BM and GBM, respectively. Therefore, strong evidence for the significance of an additional parameter in GBMBMM and its forecast capacity is established.

### Table 2: Model Performance for Estimation and Forecast

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>MAPE</th>
<th>No. of parameters</th>
<th>R2</th>
<th>$\hat{R}^2$ w.r. to B1 (F)</th>
<th>$\hat{R}^2$ w.r. to BM (F)</th>
<th>$\hat{R}^2$ w.r. to GBM (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1 +ARIMA(2,0,2)</td>
<td>2.9690</td>
<td>2.1389</td>
<td>3</td>
<td>0.999407</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>BM +ARIMA(2,0,2)</td>
<td>2.63093</td>
<td>2.10323</td>
<td>4</td>
<td>0.999877</td>
<td>0.7926 (141.40)</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>BMM +ARIMA(1,0,4)</td>
<td>2.78532</td>
<td>2.77534</td>
<td>5</td>
<td>0.999907</td>
<td>0.8432 (96.80)</td>
<td>0.2439 (11.612)</td>
<td>NA</td>
</tr>
<tr>
<td>GBM +ARIMA(2,0,3)</td>
<td>2.37584</td>
<td>3.29014</td>
<td>6</td>
<td>0.999924</td>
<td>0.8718 (79.34)</td>
<td>0.3821 (10.822)</td>
<td>NA</td>
</tr>
<tr>
<td>GBMBM +ARIMA(2,0,1)</td>
<td>2.24863</td>
<td>2.1383</td>
<td>7</td>
<td>0.999923</td>
<td>0.8702 (56.99)</td>
<td>0.3740 (6.771)</td>
<td>NA</td>
</tr>
<tr>
<td>GBMBMM +ARIMA(2,0,1)</td>
<td>1.72866</td>
<td>1.77848</td>
<td>8</td>
<td>0.999946</td>
<td>0.9089 (65.85)</td>
<td>0.5610 (14.483)</td>
<td>0.2895 (6.723)</td>
</tr>
</tbody>
</table>

6. CONCLUSIONS

Modelling a complex system is always a difficult task. Diffusion of innovation modelling in this context is facing new challenges for incorporating the interventions of new ideas, technologies and other influencing variables with a parsimonious model that helps to explain and modify the evolutionary shape of the curve with respect to time. The main aim of this paper was to compare some existing diffusion models. Considering the Bemmaor model and then extending it with modifications of the existing models considering heterogeneity levels. Because the individual propensity should not be the same for the innovator and imitator groups, it is important to consider different parameterisations to identify and validate the heterogeneity level. Therefore, the complete life cycle of the process can be studied more efficiently with minimum prediction errors.

The application of the proposed modified model in parallel to other existing models of innovation diffusion gives a fruitful comparison of the efficacy of the proposed parameter and its estimates. Results obtained from the Algerian gas production data perfectly match those of recent studies, which support the decrement trends identified by the model.
forecasts. Overall production of natural gas decreased 3% in 2011 as compared to the previous year, as British Petroleum (2012) reports.

For the first time, this study used the concept of the existence of heterogeneity among the early adopters/innovators in the diffusion process that could be further considered for multiple products’ diffusion models or diffusion models with seasonally varying time series.

REFERENCES


